THE INDICES OLYMPICS

Starter :

Given that $4^x = 9$ and $9^y = 64$, what is the value of xy?

$(4^x)^y = 64$

$4^{xy} = 4^6$

$xy = 3$

Details

- Working in pairs (in real class or on zoom)
- You must write down you working
- Calculators are not permitted!
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- Answers will be uploaded via a googleform

CORRECT +5

INCORRECT -2

1.

Write down the value of 10^{-2}.

$$10^{-2} = \frac{1}{10^2} = \frac{1}{100}$$

2.

$280 = 2^n \times 5 \times 7$

Find the value of n.

$280 = 2^n \times 5 \times 7$

$8 = 2^n$

$\therefore \underline{\underline{n = 3}}$

3.

Solve $\frac{2^9 \times 2^4}{2^n} = 2^8$

$$\frac{2^{13}}{2^n} = 2^8$$

$$2^{13-n} = 2^8$$

$$n = 5$$

4.

$5^x = 1$

Find the value of x.

$x = 0$

5.

$$\frac{(5t^3)^3 \times (2t^2)^4}{(10t^5)^2}$$

$$\frac{125t^9 \times 16t^8}{100t^{10}} = \frac{2000t^{17}}{100t^{10}}$$

$$= 20t^7$$

6.

Give your answers as fractions.

$$\left(\sqrt{\frac{3}{8}}\right)^4 = \left(\left(\frac{3}{8}\right)^{\frac{1}{2}}\right)^4$$

$$= \left(\frac{3}{8}\right)^2 = \frac{9}{64}$$

7.

Evaluate $\left(\frac{27}{343}\right)^{\frac{2}{3}}$

$$\left(\sqrt[3]{\frac{27}{343}}\right)^2 = \left(\frac{3}{7}\right)^2 = \frac{9}{49}$$

8.

Express 5^{20} as a power of 25

$$5^{20} = \left(25^{1/2}\right)^{20} = 25^{10}$$

9.

) Express $\sqrt{8}$ as a power of 2

$$\sqrt{8} = (8)^{1/2} = (2^3)^{1/2} = 2^{3/2}$$

10.

Express $\dfrac{1}{2\sqrt{2}}$ as a power of 2

$$\frac{1}{2^1 \times 2^{1/2}} = \frac{1}{2^{3/2}} = 2^{-3/2}$$

(1.

$$\frac{a\sqrt{a}}{\sqrt[3]{a^2}} = a^k$$

Work out the value of k.

$$\frac{a^1 \times a^{1/2}}{(a^2)^{1/3}} = \frac{a^{3/2}}{a^{2/3}} = a^{3/2 - 2/3}$$

$$= a^{9/6 - 4/6} = a^{5/6}$$

$$k = 5/6$$

R.

Express $\frac{1}{4\sqrt{2}}$ as a power of 2

$$\frac{1}{2^2 \times 2^{1/2}} = \frac{1}{2^{5/2}} = 2^{-5/2}$$

13.

Write $\frac{1}{4}$ as a power of 2

$$\frac{1}{4} = \frac{1}{2^2} = 2^{-2}$$

14.

$\frac{1}{1024}$ as a power of 2

$$\frac{1}{2^{10}} = 2^{-10}$$

15.

$$\left(u^a\right)^a = u^9$$

$$u^{a^2} = u^9$$

$$a^2 = 9$$

$$a = \pm 3$$

16.

$$\left(2x^3y^5\right)^3 \times \left(5x^3y^4\right)^2$$

$$= 8x^9y^{15} \times 25x^6y^8$$

$$= 200x^{15}y^{23}$$

17.

Solve $3 \times 4^{2k+8} = 24$

Show your working clearly.

$$3 \times 2^{4k+16} = 24$$

$$2^{4k+16} = 8$$

$$2^{4k+16} = 2^3$$

$$4k+16 = 3$$

$$4k = -13$$

$$k = \frac{-13}{4}$$

18.

$y = 16 \times 10^{8k}$ where k is an integer.

Find an expression, in terms of k, for $y^{\frac{5}{4}}$

Give your answer in standard form.

$(ab^2)^{11} = a^{11}b^{22}$

$y = 16 \times 10^{8k}$

$y^{\frac{5}{4}} = (16 \times 10^{8k})^{5/4}$

$= 16^{5/4} \times 10^{10k}$

$= 32 \times 10^{10k}$

$= 3.2 \times 10^{1} \times 10^{10k}$

$= 3.2 \times 10^{10k+1}$

19.

$(\sqrt{a})^7 = k\sqrt{a}$, where $k = a^n$

Find the value of n.

$$a^{\frac{7}{2}} = k\sqrt{a}$$

$$a^{\frac{7}{2}} = a^n a^{\frac{1}{2}}$$

$$a^{\frac{7}{2}} = a^{n+\frac{1}{2}}$$

$$n + \frac{1}{2} = \frac{7}{2}$$

$$n = \frac{6}{2} = 3$$

20.

Solve for n

$$2^{3n} = \frac{1}{64}$$

$$2^{3n} = \frac{1}{2^6}$$

$$2^{3n} = 2^{-6}$$

$$3n = -6$$

$$n = -2$$

21.

Simplify $\left(\frac{216x^6}{27y^3}\right)^{-\frac{2}{3}}$

$$\left(\frac{27y^3}{216x^6}\right)^{\frac{2}{3}}$$

$$\left(\sqrt[3]{\frac{27y^3}{216x^6}}\right)^2 = \left(\frac{3y}{6x^2}\right)^2 = \frac{9y^2}{36x^4}$$

$$= \frac{y^2}{4x^4}$$

22.

Express 9^{3x+2} in the form 3^y, giving y in the form $ax + b$, where a and b are constants.

$$9^{3x+2} = 3^{6x+4}$$

$a = 6$ $b = 4$

23.

Solve the equation $8^{2x-5} = 2^{x+1}$

$$2^{6x-15} = 2^{x+1}$$

$$6x - 15 = x + 1$$

$$5x = 16$$

$$x = \frac{16}{5}$$

24.

Given that $\dfrac{8^x \times 64^3}{16^7} = 32$ find x.

$$\frac{2^{3x} \times 2^{18}}{2^{28}} = 2^5$$

$$3x + 18 - 28 = 5$$

$$3x - 10 = 5$$

$$3x = 15$$

$$x = 5$$

25.

Given that $\dfrac{81^x \times 9^2}{27^4} = 1$ find x.

$$\frac{3^{4x} \times 3^4}{3^{12}} = 3^0$$

$$4x + 4 - 12 = 0$$

$$4x - 8 = 0$$

$$x = 2$$

26.

Simplify:

$$\sqrt{\frac{x^{4/3}}{x^{1/3} \times x^{8/3}}}$$

$$\left(x^{4/3 - 9/3}\right)^{1/2} = \left(x^{-5/3}\right)^{1/2} = x^{-5/6}$$

or $\dfrac{1}{x^{5/6}}$

27.

Simplify:

$$\frac{x^{\frac{5}{4}}x^{-1}}{\sqrt[4]{x^3}}$$

$$\frac{x^{1/4}}{x^{3/4}} = x^{-1/2} = \frac{1}{x^{1/2}}$$

or $\frac{1}{\sqrt{x}}$

28.

Simplify:

$$\left(\frac{x}{y}\right)^3 \times \left(\frac{y}{z}\right)^2 \times \left(\frac{z}{x}\right)^4$$

$$\frac{\cancel{x^3}}{y^{\cancel{3}}} \times \frac{\cancel{y^2}}{\cancel{z^2}} \times \frac{z^{\cancel{4}\,2}}{\cancel{x^4}\,x} = \frac{z^2}{xy}$$

29.

Solve the following equation to obtain the value of a:

$$\frac{x \times x^4 \times x^a}{\sqrt{x}} = \sqrt{\frac{\left(x^a\right)^8}{\left(\frac{1}{x^4}\right)}}$$

$$x^{a+5-\frac{1}{2}} = \left(x^{8a--4}\right)^{\frac{1}{2}}$$

$$x^{a+\frac{9}{2}} = x^{4a+2}$$

$$a+\frac{9}{2} = 4a+2$$

$$\frac{5}{2} = 3a$$

$$a = \frac{5}{6}$$

So.

Given that $\left(\sqrt{x^b}\right)^a = x^a \times x^b$ find an expression for a in terms of b:

MAKE a THE SUBJECT

$$x^{\frac{ab}{2}} = x^{a+b}$$

$$\frac{ab}{2} = a+b$$

$$ab = 2a+2b$$

$$ab - 2a = 2b$$

$$a(b-2) = 2b$$

$$a = \frac{2b}{b-2}$$

31.

Solve the following simultaneous equations to get the values of a and b:

$$\left(x^a\right)^2 = \left(x^4\right)^{2b} \qquad \text{and} \qquad \left(\sqrt{x}\right)^a = \frac{x^3}{x^b}$$

$x^{2a} = x^{8b}$

$2a = 8b$

$a = 4b$

SUB IN

$x^{a/2} = x^{3-b}$

$\frac{a}{2} = 3 - b$

$a = 6 - 2b$

$4b = 6 - 2b$

$6b = 6$

$\underline{b = 1}$

$\therefore \underline{a = 4}$